Computation of Frequency-Dependent Propagation Characteristics of Microstriplike Propagation Structures with Discontinuous Layers

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***Abstract* — A general procedure based on the method of lines is presented for the full-wave analysis of propagation structures having layered substrates with step inhomogeneities in each layer. Examples of such structures include microslab lines and microstrip near a substrate edge. It is shown that with this extended method of lines technique the frequency-dependent characteristics, including the modal impedance, current distribution, and other properties of these structures, can be calculated. To illustrate the technique typical structures such as microstrips on a finite-width dielectric slab and microstrips near a substrate edge are considered, and the effect of the proximity of the edge on the propagation characteristics of the microstrip is computed. For the case of a microstrip near a substrate edge, the numerical results obtained are compared with measured values of propagation constants.**

I. Introduction

In recent years structures consisting of layered media with step discontinuities in the dielectric constant of the layers have been proposed for application in electro-optic devices and low-loss propagation media [1]–[3]. The ability to accurately compute parameters such as the propagation constant, current distribution, and field configuration for such structures is important for a variety of applications, including the design of optimized electro-optic modulators and the design of ridge substrate and microslab waveguides [3]. In addition, these general structures can be studied to analyze the proximity effects of microstrip lines near a substrate edge. This needs to be understood to efficiently design high-packing density MMIC's [4]. A rigorous quasi-static analysis with design equations for the microstrip near the substrate edge was recently reported [5]. In this analysis the rectangular boundary division method was used to estimate substrate edge effects on microstrips for low frequencies. Preliminary results for both the quasi-static and the frequency-dependent properties of the same structures were recently reported, and a full-wave analysis of waveguide structures

Manuscript received June 14, 1989; revised September 18, 1989. This work was supported in part by Tektronix Inc., Beaverton, OR.

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IEEE Log Number 8932867.

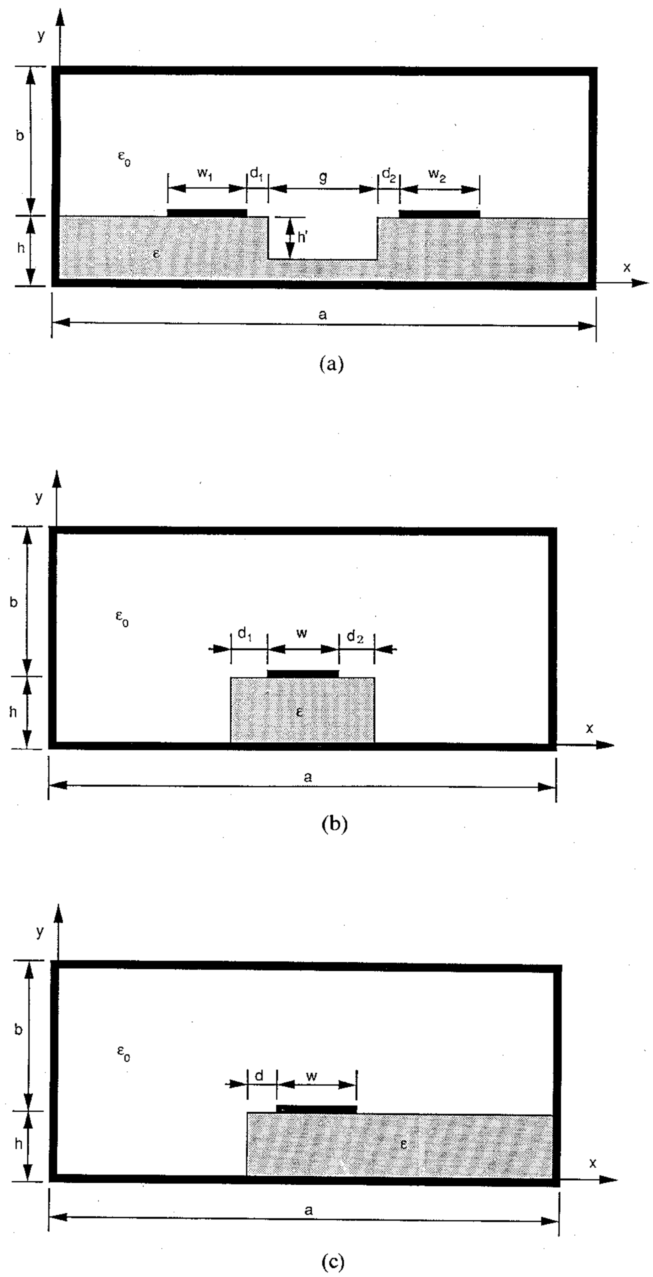

Fig. 1. Some typical structures.

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consisting of microstrips and dielectric waveguides has been presented and applied to dielectric waveguides [6]–[8]. These last methods were each different extensions of the method of lines [9]–[17].

In this paper the method of lines is extended and a general procedure is developed to analyze the class of propagation structures with layered substrates that have step discontinuities in their dielectric constants. The procedure is implemented numerically and is applied to study the frequency dependence of the propagation characteristics of structures including single and coupled microstrips, with special attention given to the role of the step inhomogeneities of the layers and their proximity to the strips. Comparisons are made with measured values of propagation constants for microstrip near a substrate edge.

II. Theory

For the class of structures considered in this paper it is convenient to choose Hertzian potentials that are x-directed so that the boundary conditions at the discontinuity can be easily enforced (Fig. 1). In each layer the fields can be written in terms of these potentials:

$$\begin{aligned}
E &= \nabla \times \nabla \times A - j\omega\mu_0 \nabla \times F \\
H &= j\omega\epsilon_r\epsilon_0 \nabla \times A + \nabla \times \nabla \times F \\
A &= \Phi^e(x, y)\exp(-j\beta z)\hat{a}_x \\
F &= \Phi^h(x, y)\exp(-j\beta z)\hat{a}_x \\
E_x &= \left(\frac{\partial^2}{\partial x^2} + \epsilon_r k_0^2\right)\Phi^e \\
H_x &= \left(\frac{\partial^2}{\partial x^2} + \epsilon_r k_0^2\right)\Phi^h \\
E_y &= \frac{\partial^2}{\partial x\,\partial y}\Phi^e - \omega\mu_0\beta\Phi^h \\
H_y &= \omega\epsilon_r\epsilon_0\beta\Phi^e + \frac{\partial^2}{\partial x\,\partial y}\Phi^h \\
E_z &= -j\beta\frac{\partial}{\partial x}\Phi^e + j\omega\mu_0\frac{\partial}{\partial y}\Phi^h \\
H_z &= -j\omega\epsilon_r\epsilon_0\frac{\partial}{\partial y}\Phi^e - j\beta\frac{\partial}{\partial x}\Phi^h
\end{aligned} \tag{1}$$

where the potentials $\phi_{e,h}$ are the solutions of the scalar Helmholtz equation:

$$\nabla_t^2\Phi^{e,h}(x, y) + (\epsilon_r k_0^2 - \beta^2)\Phi^{e,h}(x, y) = 0. \tag{2}$$

These, together with the boundary conditions at all the boundaries, lead to the formulation of the complete eigenvalue problem and the solutions. For a single substrate it is possible to divide the cross section into two regions. For problems with multiple layers the technique is readily extended and an additional region is needed for each layer. The method of lines is compatible with the implementation of the boundary conditions in both the x and y directions. Essentially it involves a discretization in the x direction and an approximation of the x derivatives followed by a similarity transformation to decouple the resulting system of ordinary differential equations. Analytical expressions for the solution of the decoupled systems are written yielding the transformed electric and magnetic Hertzian potentials as continuous functions of y. The technique is simple yet care must be taken to ensure that the boundary condition is implicitly enforced at the vertical boundaries such as the substrate edge. In general this procedure is applicable to layered structures with multiple step discontinuities. The simple case of a microstrip near the substrate edge (Fig. 1(c)) is considered in the following section to illustrate the technique.

By averaging the difference operators:

$$D_1(\Phi_i^{e,h}(y)) = \frac{1}{h_{i+1}}(\Phi_{i+1}^{e,h}(y) - \Phi_i^{e,h}(y))$$

$$D_2(\Phi_i^{e,h}(y)) = \frac{1}{h_i}(\Phi_i^{e,h}(y) - \Phi_{i-1}^{e,h}(y))$$

central difference schemes can be derived to approximate the x derivatives in the region above the substrate:

$$\frac{\partial}{\partial x}\Phi_i^{e,h}(y) = \frac{1}{2}(D_1(\Phi_i^{e,h}(y)) + D_2(\Phi_i^{e,h}(y)))$$

$$\frac{\partial^2}{\partial x^2}\Phi_i^{e,h}(y) = \frac{1}{2}(D_1(D_2(\Phi_i^{e,h}(y))) + D_2(D_1(\Phi_i^{e,h}(y)))).$$

This gives

$$\begin{aligned}
\frac{\partial}{\partial x}\Phi_i^{e,h}(y) &= \frac{-1}{2h_i}\Phi_{i-1}^{e,h}(y) + \left(\frac{1}{h_i} - \frac{1}{h_{i+1}}\right)\Phi_i^{e,h}(y) + \frac{1}{2h_{i+1}}\Phi_{i+1}^{e,h}(y) \\
\frac{\partial^2}{\partial x^2}\Phi_i^{e,h}(y) &= \frac{1}{2}\left(\frac{1}{h_i^2} + \frac{1}{h_ih_{i+1}}\right)\Phi_{i-1}^{e,h}(y) - \frac{1}{2}\left(\frac{1}{h_i^2} + \frac{2}{h_ih_{i+1}} + \frac{1}{h_{i+1}^2}\right)\Phi_i^{e,h}(y) \\
&\quad + \frac{1}{2}\left(\frac{1}{h_ih_{i+1}} + \frac{1}{h_{i+1}^2}\right)\Phi_{i+1}^{e,h}(y)
\end{aligned} \tag{3}$$

where

$$x_i = x_{i-1} + h_i$$

and

$$\Phi_i^{e,h}(y) = \Phi^{e,h}(x_i, y).$$

The difference operators are essentially the same in the substrate region; however it is necessary to write expressions for the derivatives in x which imply that at the interface of the sections the tangential electric and magnetic fields must be continuous. It is sufficient to require that across the $x = x_a$ interface the z components of E and H be continuous and the x components of B and D

be continuous. Since the substrate is nonmagnetic, these conditions are satisfied if at $x = x_a$ the scalar potentials on either side of the interface, ϕ_a and ϕ_b, satisfy

$$\begin{aligned} \epsilon_a \Phi_a^e &= \epsilon_b \Phi_b^e \\ \frac{\partial}{\partial x}\Phi_a^e &= \frac{\partial}{\partial x}\Phi_b^e \\ \Phi_a^h &= \Phi_b^h \\ \frac{\partial}{\partial x}\Phi_a^h &= \frac{\partial}{\partial x}\Phi_b^h. \end{aligned} \tag{4}$$

If the derivatives of the potentials on the lines adjacent to the interface are approximated by simple forward and backward differences, respectively, expressions for the potentials and the second derivatives of the potentials can be derived for each side of the interface by the following substitutions:

$$\begin{aligned} \frac{\partial}{\partial x}\Phi_{A-}^e &= \frac{\partial}{\partial x}\Phi_{A+}^e \\ \frac{2}{h}(\Phi_{A-}^e - \Phi_M^e) &= \frac{2}{h}(\Phi_M^e - \Phi_{A+}^e) \\ \frac{2}{h_A}(\Phi_{A-}^e - \Phi_M^e) &= \frac{2}{h_B}\left(\Phi_M^e - \frac{\epsilon_A}{\epsilon_B}\Phi_{A-}^e\right) \\ \Phi_{A-}^e &= \frac{\epsilon_b h_b}{\epsilon_A h_A + \epsilon_B h_B}\Phi_M^e + \frac{\epsilon_B h_A}{\epsilon_A h_A + \epsilon_B h_B}\Phi_{M+1}^e \\ \Phi_{A+}^e &= \frac{\epsilon_A h_B}{\epsilon_A h_A + \epsilon_B h_B}\Phi_M^e + \frac{\epsilon_A h_A}{\epsilon_A h_A + \epsilon_B h_B}\Phi_{M+1}^e. \end{aligned} \tag{5}$$

The spacing is chosen to be uniform in the neighborhood of any discontinuity in the substrate. If the spacing is chosen so that $\epsilon_A h_A = \epsilon_B h_B$ across the discontinuity, then

$$\begin{aligned} \Phi_{A-}^e &= \frac{1}{2}\Phi_M^e + \frac{1}{2}\frac{\epsilon_B}{\epsilon_A}\Phi_{M+1}^e \\ \Phi_{A-}^e &= \frac{1}{2}\frac{\epsilon_A}{\epsilon_B}\Phi_M^e + \Phi_{M+1}^e. \end{aligned} \tag{6}$$

This choice of the central differences when applied to the Helmholtz equation results in a system of ordinary differential equations with a matrix which is symmetric, and its terms in the proximity of the inhomogeneity are

$$\begin{aligned} \frac{\partial^2}{\partial x^2}\Phi_M^e &= \frac{1}{h_A^2}\Phi_{M-1}^e - \frac{2}{h_A^2}\Phi_M^e + \frac{1}{h_A h_B}\Phi_{M+1}^e \\ \frac{\partial^2}{\partial x^2}\Phi_{M+1}^e &= \frac{1}{h_A h_B}\Phi_M^e - \frac{2}{h_B^2}\Phi_{M+1}^e + \frac{1}{h_B^2}\Phi_{M+2}^e \end{aligned} \tag{7}$$

where x_m and x_{m+1} are the lines on either side of the inhomogeneity in question.

The symmetric property of the matrix resulting from this system of ordinary differential equations is important as symmetric matrices are easy to diagonalize numerically. Consequently a global change of variables is made so that the matrix remains symmetric. This is accomplished with the following substitutions:

$$\begin{aligned} \psi_i^{e,h} &= \epsilon_i \sqrt{\frac{h_i + h_{i+1}}{h_i h_{i+1}}}\,\Phi^{e,h} && \text{in lower region} \\ \psi_i^{e,h} &= \sqrt{\frac{h_i + h_{i+1}}{h_i h_{i+1}}}\,\Phi^{e,h} && \text{in upper region} \end{aligned} \tag{8}$$

and

$$\begin{aligned} \Psi^{e,h} &= T_{e,h,\mathrm{II}}^t \psi^{e,h} && \text{in lower region} \\ \Psi^{e,h} &= T_{e,h,\mathrm{I}}^t \psi^{e,h} && \text{in upper region} \end{aligned} \tag{9}$$

where the $T_{e,h}$ are the matrices of eigenvectors found after the boundary conditions on the electric walls of the box are enforced. The solution for the potential in each region is found to be

$$\begin{aligned} \Psi_{1i}^e &= A_i^1 \sinh\left(\sqrt{\Lambda_{1i}^e}\,(y-b-d)\right) && \text{(upper region)} \\ \Psi_{1i}^h &= B_i^1 \cosh\left(\sqrt{\Lambda_{1i}^h}\,(y-b-d)\right) \\ \Psi_{2i}^e &= A_i^2 \sinh\left(\sqrt{\Lambda_{2i}^e}\,y\right) && \text{(lower region)} \\ \Psi_{2i}^h &= B_i^2 \cosh\left(\sqrt{\Lambda_{2i}^h}\,y\right) \end{aligned} \tag{10}$$

where the Λ's are the eigenvalues corresponding to the respective $T_{e,h}$. With these expressions the tangential fields at the interface of the upper and lower regions may be written in matrix form by expanding equations (1).

The resulting expression for the fields and currents at the interface are found from these equations and are given as

$$\begin{aligned} \begin{bmatrix} E_z \\ jk_0/h_0 E_x \end{bmatrix} &= \begin{bmatrix} -\sqrt{\epsilon_{\text{eff}}}A_2 & B_2 \\ I & 0 \end{bmatrix} \\ &\cdot \begin{bmatrix} -\sqrt{\epsilon_{\text{eff}}}(A_1 - A_2) & (B_1 - B_2) \\ -(C_1 - C_2) & -\sqrt{\epsilon_{\text{eff}}}(F_1 - F_2) \end{bmatrix}^{-1} \\ &\cdot \begin{bmatrix} B_1 & 0 \\ -\sqrt{\epsilon_{\text{eff}}}F_1 & I \end{bmatrix} \begin{bmatrix} jk_0/h_0 J_z \\ J_x \end{bmatrix} \end{aligned} \tag{11}$$

where

$$\begin{aligned} A_1 &= D_{\mathrm{I}}^e r_1 T_{e\mathrm{I}} \Lambda_{e\mathrm{I}}^{-1} T_{e\mathrm{I}}^t r_1^{-1} \\ A_2 &= D_{\mathrm{II}}^e r_2 T_{e\mathrm{II}} \Lambda_{e\mathrm{II}}^{-1} T_{e\mathrm{II}}^t r_2^{-1} \\ B_1 &= r_1 T_{h\mathrm{I}} \gamma_{h\mathrm{I}} \Lambda_{h\mathrm{I}}^{-1} T_{h\mathrm{I}}^t r_1^{-1} \\ B_2 &= r_1 T_{h\mathrm{II}} \gamma_{h\mathrm{II}} \Lambda_{h\mathrm{II}}^{-1} T_{h\mathrm{II}}^t r_1^{-1} \\ C_1 &= r_1 T_{e\mathrm{I}} \gamma_{e\mathrm{I}} \Lambda_{e\mathrm{I}}^{-1} T_{e\mathrm{I}}^t r_1^{-1} \\ C_2 &= \epsilon_r r_2 T_{e\mathrm{II}} \gamma_{e\mathrm{II}} \Lambda_{e\mathrm{II}}^{-1} T_{e\mathrm{II}}^t r_2^{-1} \\ F_1 &= D^h r_1 T_{h\mathrm{I}} \Lambda_{h\mathrm{I}}^{-1} T_{h\mathrm{I}}^t r_1^{-1} \\ F_2 &= D^h r_1 T_{h\mathrm{II}} \Lambda_{h\mathrm{II}}^{-1} T_{h\mathrm{II}}^t r_1^{-1} \end{aligned} \tag{12}$$

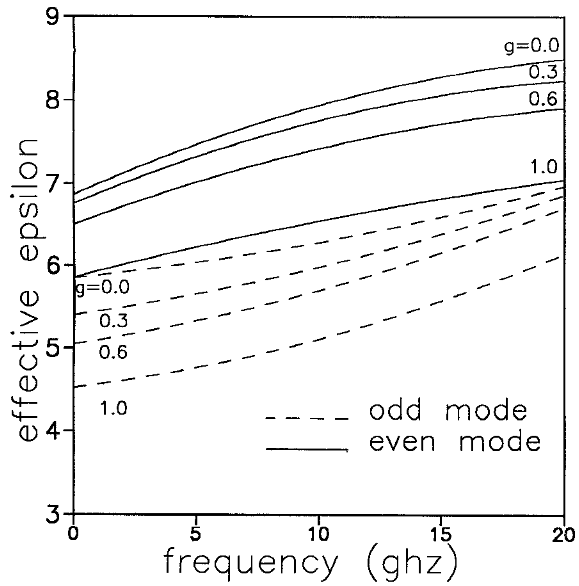

Fig. 2. The even- and odd-mode effective dielectric constants as a function of frequency for the structure of Fig. 1(a): $w = 1$ mm, $h = 1$ mm, $\epsilon_r = 9.8$, the strips are separated by 1 mm, and g is in mm. Box dimensions are $a = 10$ mm, $b = 100$ mm.

D_{I}^e, D_{II}^e, and D^h are the first-order difference operators; the $r_{1,2}$ are the normalization factors defined in (8); and the $\gamma_{e,h,\text{I,II}}$ are defined by the equation

$$\frac{\partial}{\partial y}\Psi_{\text{I,II},i}^{e,h} = \gamma_{e,h,\text{I,II},i}\Psi_{\text{I,II},i}^{e,h}.$$

A reduced matrix can be formed by considering only those lines which pass through a metallic strip. After enforcing the boundary conditions a determinantal equation results whose solution leads to the propagation constant which can be used to compute the currents and the fields:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} E_z \\ jk_0/h_0 E_x \end{bmatrix}_{\text{ON STRIP}} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} jk_0/h_0 J_z \\ J_x \end{bmatrix}_{\text{ON STRIP}} \tag{13}$$

The line impedance is calculated from the power–current definition as given by

$$Z = \frac{\frac{1}{2}\int E \times H^* \cdot \hat{a}_z \, dx\, dy}{\int J_z \cdot dx\, dy}.$$

The expressions for power are given in the Appendix.

III. Results

The propagation characteristics of typical structures shown in Fig. 1 were computed. The computations were made in a variety of machines including IBM-386 compatible personal computers. Typical execution times for this class of structures were 15 min. The program was checked by computing the results for known cases of single and coupled microstrips on a homogeneous layer.

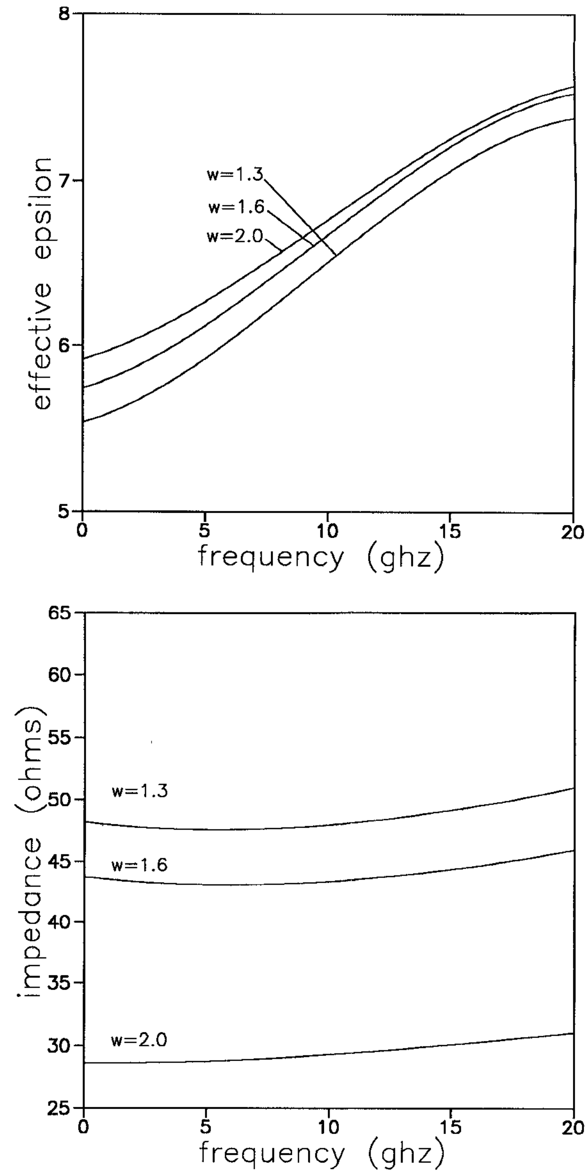

Fig. 3. Propagation characteristics of a microstrip on a finite-width dielectric slab of Fig. 1(b) as a function of frequency. (a) Effective dielectric constant. (b) Characteristic impedance. $D = 2$ mm, $\epsilon_r = 9.8$, $h = 1$ mm, $a = 10$ mm, $b = 100$ mm, and w is in mm.

In Fig. 2 both the even-mode and odd-mode propagation constants for the proposed electro-optic modulatorlike structure designed to increase the phase velocity are shown for a variety of gap sizes. Fig. 3 shows the effective dielectric constants and modal impedances for a selection of strips of different widths on a finite-width dielectric slab. The effect of the proximity of the edge of the substrate on the propagation constant of a nominal 50 Ω line is demonstrated as a function of frequency in Fig. 4.

In Fig. 5, calculations are compared with standard resonance measurements of the effective dielectric constant for microstrip lines near the substrate edge. The structures were fabricated on a 50 mil substrate with $\epsilon_r = 10.2$ having the first resonance around 2 GHz. It is seen that the proximity effect due to the edge for these structures was predicted accurately (to within 1 percent).

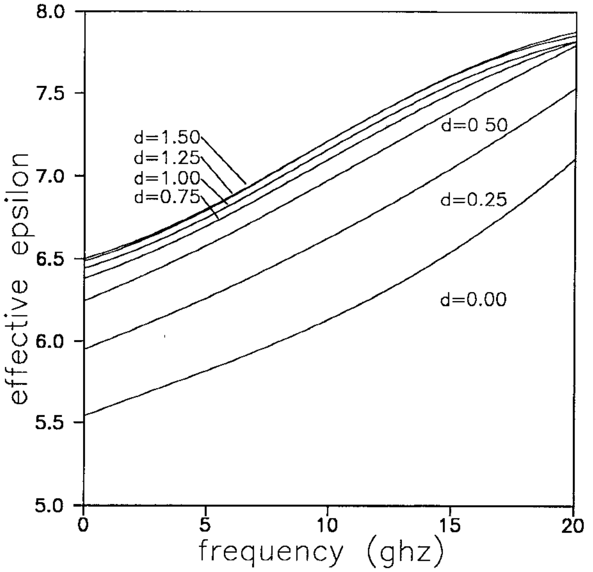

Fig. 4. Frequency-dependent effective dielectric constant of a microstrip near a substrate edge. $w = 1$ mm, $\epsilon_r = 9.8$, $h = 1$ mm, $a = 10$ mm, $b = 100$ mm, and d is in mm.

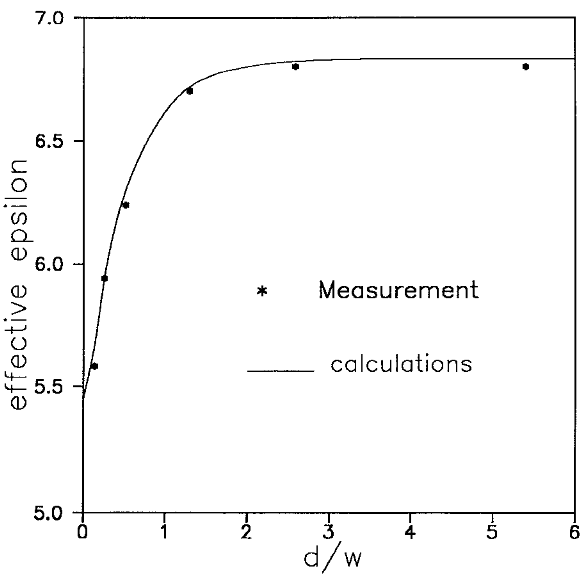

Fig. 5. Effective dielectric constant of a microstrip as a function of proximity to the substrate edge. $w = 0.925$ mm, $h = 1.27$ mm, $\epsilon_r = 10.2$, and frequency $\simeq 2$ GHz.

IV. Conclusions

The method of lines has been generalized to compute the propagation characteristics of a class of propagation structures with layered substrates having step inhomogeneities. The technique accurately predicts the effects of the step inhomogeneity in the substrate on the propagation characteristics of the structures. As a result this algorithm should prove useful for a host of problems, among them the analysis of microstrip lines which approach a substrate edge or the design of such structures as ridge substrate, microslab waveguides, and electro-optic modulators.

Appendix
Expressions for Power

$$P_1 = -\sqrt{\epsilon_{\text{eff}}}\, k_0^2 h_0^2 R_{\text{I}}^t [I_{\text{swsI}}] \Lambda_{e\text{I}}^{-1} R_{\text{I}} - R_{\text{I}}^t [I_{\text{susI}}] \gamma_{\text{I}}^h \Lambda_{h\text{I}}^{-1} Q_{\text{I}} - R_{\text{I}}^t \Lambda_{e\text{I}}^{-1} \gamma_{e\text{I}} [I_{\text{cvcI}}] Q_{\text{I}} + \sqrt{\epsilon_{\text{eff}}}\, k_0^2 h_0^2 Q_{\text{I}}^t \Lambda_{h\text{I}}^{-1} [I_{\text{cxcI}}] Q_{\text{I}}$$

$$P_2 = -\sqrt{\epsilon_{\text{eff}}}\, k_0^2 h_0^2 R_{\text{II}}^t [I_{\text{swsII}}] \Lambda_{e\text{II}}^{-1} R_{\text{II}} - R_{\text{II}}^t [I_{\text{susII}}] \gamma_{\text{II}}^h \Lambda_{h\text{II}}^{-1} Q_{\text{II}} - R_{\text{II}}^t \Lambda_{e\text{II}}^{-1} \gamma_{e\text{II}} [I_{\text{cvcII}}] Q_{\text{II}} + \sqrt{\epsilon_{\text{eff}}}\, k_0^2 h_0^2 Q_{\text{II}}^t \Lambda_{h\text{II}}^{-1} [I_{\text{cxcII}}] Q_{\text{II}}$$

with

$$W_{\text{I}} = T_{e\text{I}}^t r_{e\text{I}} w r_{e\text{I}} T_{e\text{I}} \qquad W_{\text{II}} = T_{e\text{II}}^t r_{e\text{II}} w \epsilon_r r_{e\text{II}} T_{e\text{II}}$$

$$U_{\text{I}} = T_{e\text{I}}^t r_{e\text{I}} w D_3 r_h T_{h\text{I}} \qquad U_{\text{II}} = T_{e\text{II}}^t r_{e\text{II}} w D_3 r_h T_{h\text{II}}$$

$$R_{\text{I}} = E_{x\text{I}}^t r_{e\text{I}} T_{e\text{I}} \qquad R_{\text{II}} = E_{x\text{II}}^t r_{e\text{II}} T_{e\text{II}}$$

$$Q_{\text{I}} = \eta H_{x\text{I}}^t r_h T_{h\text{I}} \qquad Q_{\text{II}} = \eta H_{x\text{II}}^t r_h T_{h\text{II}}$$

$$V_{\text{I}} = T_{e\text{I}}^t r_{e\text{I}} D_1^t w r_h T_{h\text{I}} \qquad V_{\text{II}} = T_{e\text{II}}^t r_{e\text{II}} D_2^t w r_h T_{h\text{II}}$$

$$X_{\text{I}} = T_{h\text{I}}^t r_h w r_h T_{h\text{I}} \qquad X_{\text{II}} = T_{h\text{II}}^t r_h w r_h T_{h\text{II}}$$

and

$$[I_{\text{swsI}}]_{ij} = h_0 w_{\text{I}_{ij}} \left(\sqrt{\Lambda_{\text{I}_i}} \coth \sqrt{\Lambda_{\text{I}_i}} \frac{b}{h_0} \right) \Big/ (\Lambda_{\text{I}_i} - \Lambda_{\text{I}_j}) - h_0 w_{\text{I}_{ij}} \left(\sqrt{\Lambda_{\text{I}_j}} \coth \sqrt{\Lambda_{\text{I}_j}} \frac{b}{h_0} \right) \Big/ (\Lambda_{\text{I}_i} - \Lambda_{\text{I}_j})$$

$$[I_{\text{susI}}]_{ij} = h_0 u_{\text{I}_{ij}} \left(\sqrt{\Lambda_{\text{I}_i}} \coth \sqrt{\Lambda_{\text{I}_i}} \frac{b}{h_0} \right) \Big/ (\Lambda_{\text{I}_i} - \Lambda_{\text{I}_j}) - h_0 u_{\text{I}_{ij}} \left(\sqrt{\Lambda_{\text{I}_j}} \coth \sqrt{\Lambda_{\text{I}_j}} \frac{b}{h_0} \right) \Big/ (\Lambda_{\text{I}_i} - \Lambda_{\text{I}_j})$$

$$[I_{\text{cvcI}}]_{ij} = h_0 v_{\text{I}_{ij}} \left(\sqrt{\Lambda_{\text{I}_i}} \tanh \sqrt{\Lambda_{\text{I}_i}} \frac{b}{h_0} \right) \Big/ (\Lambda_{\text{I}} - \Lambda_{\text{I}_j}) - h_0 v_{\text{I}_{ij}} \left(\sqrt{\Lambda_{\text{I}_j}} \tanh \sqrt{\Lambda_{\text{I}_j}} \frac{b}{h_0} \right) \Big/ (\Lambda_{\text{I}_i} - \Lambda_{\text{I}_j})$$

$$[I_{\text{cxcI}}]_{ij} = h_0 x_{\text{I}_{ij}} \left(\sqrt{\Lambda_{\text{I}_i}} \tanh \sqrt{\Lambda_{\text{I}_i}} \frac{b}{h_0} \right) \Big/ (\Lambda_{\text{I}_i} - \Lambda_{\text{I}_j}) - h_0 x_{\text{I}_{ij}} \left(\sqrt{\Lambda_{\text{I}_j}} \tanh \sqrt{\Lambda_{\text{I}_j}} \frac{b}{h_0} \right) \Big/ (\Lambda_{\text{I}_i} - \Lambda_{\text{I}_j})$$

$$[I_{\text{swsII}}]_{ij} = h_0 w_{\text{II}_{ij}} \left(\sqrt{\Lambda_{\text{II}_i}} \coth \sqrt{\Lambda_{\text{II}_i}} \frac{d}{h_0} \right) \Big/ (\Lambda_{\text{II}_i} - \Lambda_{\text{II}_j}) - h_0 w_{\text{II}_{ij}} \left(\sqrt{\Lambda_{\text{II}_j}} \coth \sqrt{\Lambda_{\text{II}_j}} \frac{d}{h_0} \right) \Big/ (\Lambda_{\text{II}_i} - \Lambda_{\text{II}_j})$$

$$[I_{\text{susII}}]_{ij} = h_0 u_{\text{II}_{ij}} \left(\sqrt{\Lambda_{\text{II}_i}} \coth \sqrt{\Lambda_{\text{II}_i}} \frac{d}{h_0} \right) \Big/ (\Lambda_{\text{II}_i} - \Lambda_{\text{II}_j}) - h_0 u_{\text{II}_{ij}} \left(\sqrt{\Lambda_{\text{II}_j}} \coth \sqrt{\Lambda_{\text{II}_j}} \frac{d}{h_0} \right) \Big/ (\Lambda_{\text{II}_i} - \Lambda_{\text{II}_j})$$

$$[I_{\text{cvcII}}]_{ij} = h_0 v_{\text{I}_{ij}} \left(\sqrt{\Lambda_{\text{II}_i}} \tanh \sqrt{\Lambda_{\text{II}_i}} \frac{d}{h_0} \right) \Big/ (\Lambda_{\text{II}_i} - \Lambda_{\text{II}_j}) - h_0 v_{\text{II}_{ij}} \left(\sqrt{\Lambda_{\text{II}_j}} \tanh \sqrt{\Lambda_{\text{II}_j}} \frac{d}{h_0} \right) \Big/ (\Lambda_{\text{II}_i} - \Lambda_{\text{II}_j})$$

$$[I_{\text{cxcII}}]_{ij} = h_0 x_{\text{II}ij}\left(\sqrt{\Lambda_{\text{II}i}}\tanh\sqrt{\Lambda_{\text{II}i}}\,\frac{d}{h_0}\right)\Big/(\Lambda_{\text{II}i}-\Lambda_{\text{II}j}) - h_0 x_{\text{II}ij}\left(\sqrt{\Lambda_{\text{II}j}}\tanh\sqrt{\Lambda_{\text{II}j}}\,\frac{d}{h_0}\right)\Big/(\Lambda_{\text{II}i}-\Lambda_{\text{II}j}).$$

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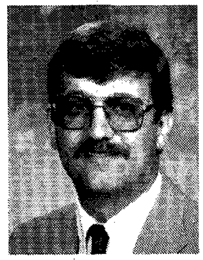

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